Fuzzy Logic based Logical Query Answering on Knowledge Graphs

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Overview

We present **FuzzQE**, a fuzzy logic based query embedding framework for answering First-Order Logic (FOL) queries over KGs.

- FuzzQE follows product logic to define logical operators in a principled and learning free manner.
- Extensive experiments on two benchmark datasets demonstrate that FuzzQE achieves significantly better performance in answering FOL queries compared to the state-of-the-art methods.
- When trained with only KG link prediction, FuzzQE can achieve comparable performance with the systems trained with all FOL queries. This is a huge advantage in real-world applications, since complex FOL training queries are often arduous to collect and not available in most real-world KGs.

Logic Laws and Desired Logical Query Answering Model Properties

$\phi(q, e)$ estimates the probability that entity e answers query q

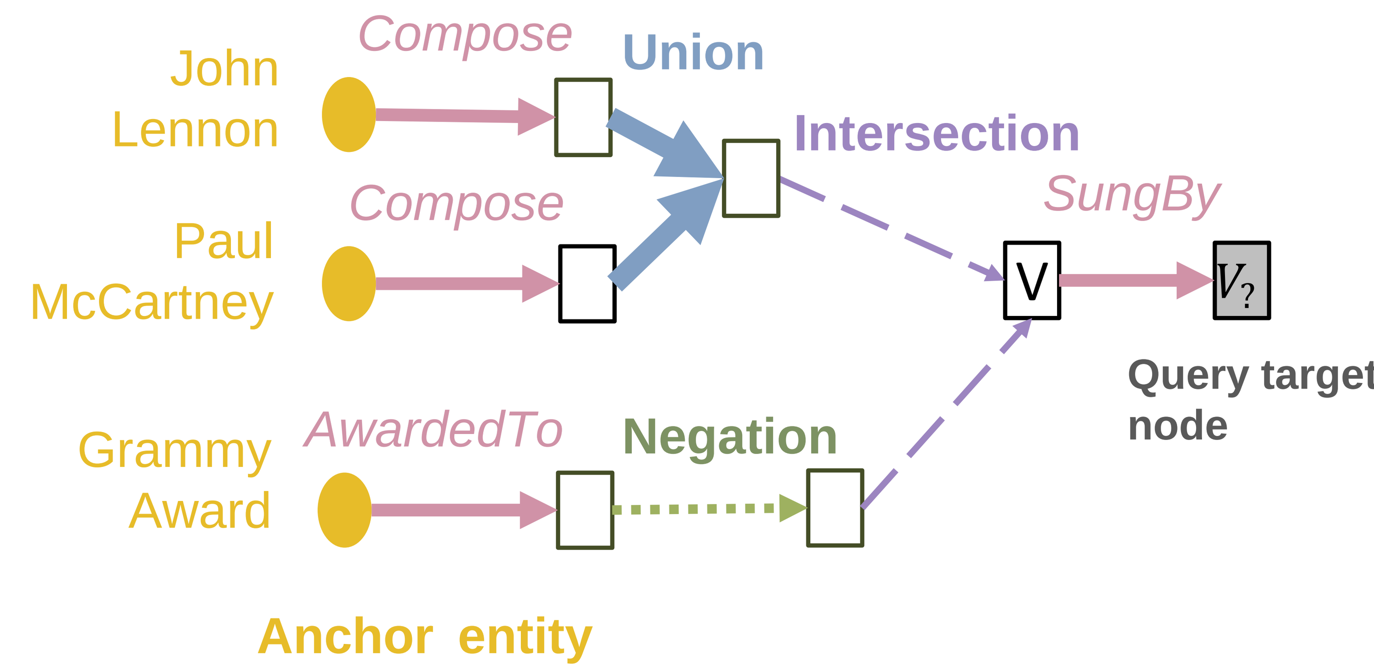
Axioms and derived logic laws in both classic logic and fuzzy logic

Desired model property according to the logic law

	Logic Law	Model Property
$\wedge$	Conjunction elimination $\varphi \wedge \psi \rightarrow \varphi$ $\varphi \wedge \psi \rightarrow \psi$	 $\phi(q_1 \wedge q_2, e) \leq \phi(q_1, e)$ $\phi(q_1 \wedge q_2, e) \leq \phi(q_2, e)$
	Commutativity $\varphi \wedge \psi \leftrightarrow \psi \wedge \varphi$	 $\phi((q_1 \wedge q_2), e) = \phi((q_2 \wedge q_1), e)$
	Associativity $(\varphi \wedge \psi) \wedge \chi \leftrightarrow$ $\varphi \wedge (\psi \wedge \chi)$	 $\phi((q_1 \wedge q_2) \wedge q_3, e)$ $= \phi(q_1 \wedge (q_2 \wedge q_3), e)$
$\vee$	Disjunction amplification $\varphi \rightarrow \varphi \vee \psi$ $\varphi \rightarrow \varphi \vee \varphi$	 $\phi(q_1 \vee q_2, e) \geq \phi(q_1, e)$ $\phi(q_1 \vee q_2, e) \geq \phi(q_2, e)$
	Commutativity $\varphi \vee \psi \leftrightarrow \psi \vee \varphi$	 $\phi((q_1 \vee q_2), e) = \phi((q_2 \vee q_1), e)$
	Associativity $(\varphi \vee \psi) \vee \chi \leftrightarrow$ $\varphi \vee (\psi \vee \chi)$	 $\phi((q_1 \vee q_2) \vee q_3, e)$ $= \phi(q_1 \vee (q_2 \vee q_3), e)$
$\neg$	Involution $\neg\neg\varphi \rightarrow \varphi$	 $\phi(q, e) = \phi(\neg\neg q, e)$
	Non-contradiction $\varphi \wedge \neg\varphi \rightarrow \overline{0}$	 $\phi(q, e) \uparrow \;\Rightarrow\; \phi(\neg q, e) \downarrow$

First-Order Logic (FOL) Queries

$$q = V_?: \exists V \quad (Compose(\text{John Lennon}, V) \vee Compose(\text{Paul McCartney}, V)) \wedge \neg AwardedTo(\text{Grammy Award}, V) \wedge SungBy(V, V_?)$$



FuzzQE

- **Relation Projection**

$$q = \sigma(W_r e + b_r) \qquad \mathbf{W}_r = \sum_{i=1}^{m} a_{ri} \mathbf{M}_i$$

- **Fuzzy Logic based Logic Operators**

Product Logic

A fuzzy logic system developed with the classical logic axiom and two extra axioms. Satisfy all the listed logic laws.

$$q_1 \wedge q_2: \quad \mathcal{C}(q_1, q_2) = q_1 \circ q_2$$
$$q_1 \vee q_2: \quad \mathcal{D}(q_1, q_2) = q_1 + q_2 - q_1 \circ q_2$$
$$\neg q: \quad \mathcal{N}(q) = 1 - q$$

- **Loss Function**

$$L = -\log \sigma(\phi(q, e) - \gamma) - \sum_{j=1}^{n} \frac{1}{n} \log \sigma(\gamma - \phi(q, e'))$$

Comparison with Existing Works on Desired Model Properties

- Proposition 1. Our conjunction operator $\mathcal{C}$ is commutative, associative, and satisfies conjunction elimination.
- Proposition 2. Our disjunction operator $\mathcal{D}$ is commutative, associative, and satisfies disjunction amplification.
- Proposition 3. Our negation operator $\mathcal{N}$ is involutory and satisfies non-contradiction.

	$\wedge$				$\vee$				$\neg$		
	Expressivity (Closed)	*Com.*	*Asso.*	*Elim.*	Expressivity (Closed)	*Com.*	*Asso.*	*Ampli.*	Expressivity (Closed)	*Inv.*	*Non-Contra.*
GQE	✓(✓)	✓	✓	✗	✓(✗)	✓	✓	✓	✗	N/A	N/A
Query2Box	✓(✓)	✓	✓	✓	✓(✗)	✓	✓	✓	✗	N/A	N/A
$\text{BetaE}_{\text{DNF}}$	✓(✓)	✓	✓	✗	✓(✗)	✓	✓	✓	✓(✓)	✓	✗
BetaE_{DM}	✓(✓)	✓	✓	✗	✓(✓)	✓	✓	✗	✓(✓)	✓	✗
FuzzQE	✓(✓)	✓	✓	✓	✓(✓)	✓	✓	✓	✓(✓)	✓	✓

Our model satisfies all these desired properties!

Experiments

- **Train with Complex Queries**

Model	avg_p	avg_n	1p	2p	3p	2i	3i	pi	ip	2u	up	2in	3in	inp	pin	pni
FB15k-237																
GQE	16.3	-	35.0	7.2	5.3	23.3	34.6	16.5	10.7	8.2	5.7	-	-	-	-	-
Query2Box	20.1	-	40.6	9.4	6.8	29.5	42.3	21.2	12.6	11.3	7.6	-	-	-	-	-
BetaE	20.9	5.5	39.0	10.9	10.0	28.8	42.5	22.4	12.6	12.4	9.7	5.1	7.9	7.4	3.5	3.4
FuzzQE	**24.0**	**7.8**	**42.8**	**12.9**	**10.3**	**33.3**	**46.9**	**26.9**	**17.8**	**14.6**	**10.3**	**8.5**	**11.6**	**7.8**	**5.2**	**5.8**

- **Train with only Link Prediction**

Model	avg_p	avg_n	1p	2p	3p	2i	3i	pi	ip	2u	up	2in	3in	inp	pin	pni
FB15k-237																
GQE	17.7	-	41.6	7.9	5.4	25.0	33.6	16.3	10.9	11.9	6.2	-	-	-	-	-
Query2Box	18.2	-	42.6	6.9	4.7	27.3	36.8	17.5	11.1	11.7	5.5	-	-	-	-	-
BetaE	19.0	0.4	**53.1**	6.0	3.9	32.0	37.7	15.8	8.5	10.1	3.5	0.1	1.4	0.1	0.1	0.1
FuzzQE	**21.9**	**6.6**	44.0	**10.8**	**8.6**	**32.3**	**41.4**	**22.7**	**15.1**	**13.5**	**8.7**	**7.7**	**9.5**	**7.0**	**4.1**	**4.7**

References

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